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Passive Control of the Vibration of Flooring Systems using a Gravity Compensated Non-Linear Energy Sink

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ABSTRACT
This paper investigates a new type of non-linear energy sink (NES) proposed to mitigate vibrations induced on flooring systems. An NES is a type of passive mass damper which includes an essentially non-linear stiffness component which allows these devices to mitigate vibrations across a broad range of frequencies. Flooring systems are subjected to a wide range of changing frequencies resulting from their use and occupancy. Human or mechanical induced vibrations can cause serviceability issues and, in some cases, damage to flooring systems. Single vertical floor vibration modes have been mitigated with traditional linear passive mass dampers. While these devices can be effective, they are limited because they are tuned to a specific frequency and are ineffective when the response occurs at different frequencies or changing system properties shift the frequency of the mode they are designed to mitigate. NESs have been studied and shown to be effective in horizontal vibration applications; however, in vertical applications, gravity produces an offset in the NES which reduces their effectiveness at mitigating vertical loads. This paper proposes a geometric mechanism for producing a NES that compensates for the effect of gravity while maintaining the NES's required cubic non-linear stiffness relationship. A simplified model of a flooring system coupled with a gravity compensated NES (GCNES) is developed and analysed. The results of this study demonstrate that the GCNES can be effective at mitigating the response of inputs at the floor's primary response frequency as well as higher input frequencies.

KEYWORDS: *floor vibration, mass damper, nonlinear energy sink, passive control*

1. INTRODUCTION

Floor systems, like other structural components, are constantly subjected to dynamic loads. If the loading is large or the loading frequency resonates with any of the floor system's natural frequencies, serviceability issues related to excessive vibrations of the flooring system can result. These serviceability issues can range from vibrations that become a nuisance to occupants to vibrations that interfere with high precision equipment. Furthermore, increases in the use of lightweight materials, long-spans, and open-plan floor systems have led to an increase in flooring systems that are naturally more susceptible to vibration concerns. Excitation from sources such as dancing, walking, running, and machinery can lead to resonance problems and are becoming more widespread in multi-story structures in populated areas (Allen and Pernica 1998; Saidi et al. 2006).

Traditionally, the primary way to avoid vibration problems in floors has been to avoid resonance whenever possible. At a basic level, this can be accomplished by moving the excitation source to ground level or a more suitable location (Allen and Pernica 1998). When this is not possible, to reduce the vibration of a floor, changes to the mass and stiffness of a flooring system can be made such that the flooring system's modal properties move to different frequency regimes. Current building codes and guidelines try to take into account the physical properties of structural systems, but, arguably, do not model frequency loading induced by human activity very accurately (Ebrahimpour and Sack 2005); thus, modifying the mass or stiffness of a flooring system to avoid these input frequencies is not always successful. Additionally, changes to the structure or mass of a flooring system, particularly in reuse or retrofit scenarios, can be prohibitively costly.

Alternatively, supplemental devices designed to mitigate vertical vibrations have been studied and implemented. One important type of passive device for vertical floor vibration mitigation is the tuned mass damper (TMD). The TMD has been successfully used for vibration mitigation, but faces limitations. One major issue related to TMDs is that they are tuned to a particular frequency; thus, they have a limited range of frequencies in which they are effective. TMDs must be tuned to mitigate the resonant frequency of concern and multiple differing TMDs are needed to mitigate multiple resonant frequencies (Webster and Vaicaitis 1992). TMDs are also susceptible to detuning caused by issues such as fatigue, damage, and temperature variations which can result in changes in the natural frequencies of the flooring system or TMD (Roffel et al. 2010).

A proposed passive alternative, discussed in this paper, is called a nonlinear energy sink (NES). A NES is a type of mass damper, but unlike the TMD, incorporates an essentially nonlinear stiffness component (Lee et al. 2008).

In most cases, this stiffness component has a cubic relationship between restoring force and relative displacement. This elastic and repeatable stiffness component, usually produced with a geometric nonlinearity, allows the NES to engage in targeted energy transfer (TET). TET describes an irreversible energy capturing response that occurs from the primary system to the attached NES. Furthermore, the NES has no preferred vibration frequency, giving it a broad operating frequency regime (McFarland et al. 2005).

The main problem when realizing a vertical NES is the effect of gravity. The vertical gravitational force on the mass of the NES produces an offset in the at rest position of the device; the device will displace downward due to the weight of the NES. For a TMD, this offset is generally not an issue because the stiffness of the TMD is constant. For the NES, this offset displacement results in the at rest point being moved from a position where the device has no tangent stiffness to a position where the device has a non-zero tangent stiffness. This can have a large effect on the dynamics of the NES because, instead of possessing a strong non-linearizable nonlinearity, the device will operate like a perturbed linear oscillator at some displacement levels. In order to avoid this, the restoring force provided by the NES must take this into account and compensate for the effect of gravity.

In this paper, a NES is proposed using a restoring force mechanism, which was originally studied for application in a vertical vibration isolator (Carrella et al. 2007; Kovacic et al. 2008), that compensates for the effect of gravity while maintaining the required dynamic non-linear stiffness component. A simplified flooring system, which is modelled as a beam, that has an attached GCNES is analysed. An optimization is performed to determine the GCNES's optimum damping and stiffness parameters. The response of this system with a GCNES is then compared to the floor's response with an optimized TMD and without any control device.

2. PROPOSED GRAVITY COMPENSATED DEVICE (GCNES)

The proposed device, which was previously utilized in vibration isolation studies (Carrella et al. 2007; Kovacic et al. 2008), produces a quasi-zero stiffness about its at rest position. This device produced the low dynamic stiffness needed in vibration isolation and high static stiffness needed to account for static deflection. This geometric configuration is studied in this paper and is optimized, with the goal of vibration mitigation of the structure it is attached to, in order to produce the desired cubic non-linear stiffness while accounting and compensating for the effects of gravity that is necessary in an NES application. A system schematic is shown in Fig. 1 where k_1 is the inline spring constant, k_2 is the oblique spring constant, L_0 is the unstretched oblique spring length, a is the constant horizontal oblique springs' component length, h is the vertical oblique springs' component length, y is the vertical position of the NES mass relative to the unstretched position, and θ is the angle of the oblique springs relative to the horizontal.

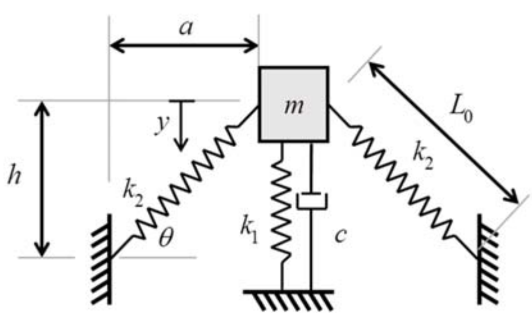

Figure 1 Proposed GCNES device schematic

Expressions for L_0 and θ are found in Eq. 1 and Eq. 2, respectively.

$$L_0 = \sqrt{a^2 + h^2} \tag{1}$$

$$\theta = \tan^{-1}\left(\frac{h-y}{a}\right) \tag{2}$$

2.1 Restoring force derivation

A free body diagram showing the resulting forces acting on the NES mass is depicted in Fig. 2 where F_1 represents the total inline spring force and F_2 represents the total oblique spring force. For the sake of simplicity and to focus on the nonlinear restoring force from the springs, the restoring force from the damper is neglected in this figure and for the remainder of the derivation. The total resulting restoring force in the vertical direction, the y direction in the figure, can be expressed in Eq. 3.

$$F_s = F_1 + 2F_2 \sin(\theta)$$
$$F_s = k_1 y + 2k_2\left(L_0 - \sqrt{a^2 + (h-y)^2}\right)\sin(\theta) \tag{3}$$

Combining Eq. 2 and Eq. 3 yields

$$F_s = k_1 y + 2k_2\left(L_0 - \sqrt{a^2 + (h-y)^2}\right)\sin\left(\tan^{-1}\left(\frac{h-y}{a}\right)\right)$$
$$F_s = k_1 y + 2k_2\left(L_0 - \sqrt{a^2 + (h-y)^2}\right)\frac{(h-y)}{a\sqrt{a^2 + (h-y)^2}} \tag{4}$$

This expression can be simplified by performing a change of variables where $\overline{y} = y - h$ in Eq. 5.

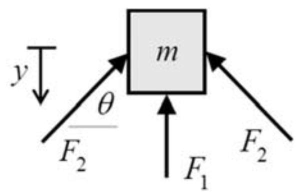

Figure 2 Free body diagram of the GCNES

A Taylor series expansion is performed about the point $\overline{y} = 0$ to obtain an approximation of the restoring force (Eq. 6). At the point $\overline{y} = 0$, the oblique springs are horizontal, as depicted in Fig. 3, and in a state of compression.

$$F_s = k_1\left(\overline{y} + h\right) - 2k_2\left(L_0 - \sqrt{a^2 + \overline{y}^2}\right)\frac{\overline{y}}{\sqrt{a^2 + \overline{y}^2}} \tag{5}$$

$$F_{s-approx} = hk_1 + \left(k_1 + 2k_2\frac{a - L_0}{a}\right)\overline{y} - k_2\left(\frac{a - L_0}{a^3} - \frac{1}{a^2}\right)\overline{y}^3 + H.O.T. \tag{6}$$

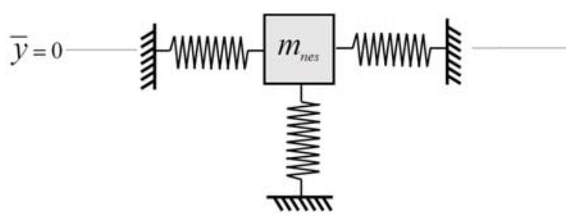

Figure 3 Physical expansion point

In order for the restoring force to possess only a constant and cubic stiffness term, the coefficient on the linear term of the restoring force must be zero; therefore, Eq. 7 must be satisfied.

$$k_1 + 2k_2\frac{a - L_0}{a} = 0 \tag{7}$$

For the system to be vibrating about the point $\overline{y} = 0$, $\overline{y} = 0$ must be the static deflection point resulting from the effect of gravity on the mass of the device. To satisfy this, Eq. 8 must be true.

$$F_s\left(\overline{y}=0\right)=mg=hk_1 \tag{8}$$

Additionally, for the system to possess the desired cubic stiffness, which might be previously determined from an optimization analysis, Eq. 9 must be satisfied.

$$k_{nes}=-k_2\left(\frac{a-L_0}{a^3}-\frac{1}{a^2}\right) \tag{9}$$

In order to solve for the necessary unknown parameters (k_1 , k_2 , a , and h), Eq. 7, Eq. 8, and Eq. 9 are solved simultaneously with one parameter set to a convenient value.

3. FLOOR MODEL

For simplicity, a floor slab modelled as a simply supported beam is considered. The dead load from normal weight concrete and a general live load associated with public use facilities are imposed on the beam. The resulting mass of the beam considered in the analysis is 3.556 slug/in . The modulus of elasticity and moment of inertia of the beam considered in the analysis are 4031000 lb/in^2 and 4200 in^4, respectively. The damping in this system was set to correspond to 1% damping in each mode of the beam without any control device attached.
Fig. 4 depicts the model coupled with the GCNES device consisting of a linear damper and cubic stiffness element. In this figure, x_p is the loading position, p(t) is the load, x_{nes} is the GCNES position, L is the span length, m_{nes} is the GCNES mass, $u(x,t)$ is the beam displacement, and x(t) is the GCNES displacement. For this analysis, L=144 in , $x_p = 36$ in , and $x_{nes} = 108$ in . The GCNES mass, as well as the mass of a TMD located at the same point for comparison purposes is set to 5% of the mass of the flooring system. Additionally, a case will be considered where no device, and no additional mass, is present. Furthermore, the response of the beam at $x = 36$ in is primarily considered for the remainder of the paper.
A cubic stiffness is considered in the model of the GCNES used this analysis. This is done based on the assumption that the GCNES compensated restoring force can be produced and the static effect of gravity will not affect the dynamics of the compensated device. In a later section, the optimized nonlinear stiffness value used will be demonstrated to be dynamically achievable with the full GCNES restoring force.

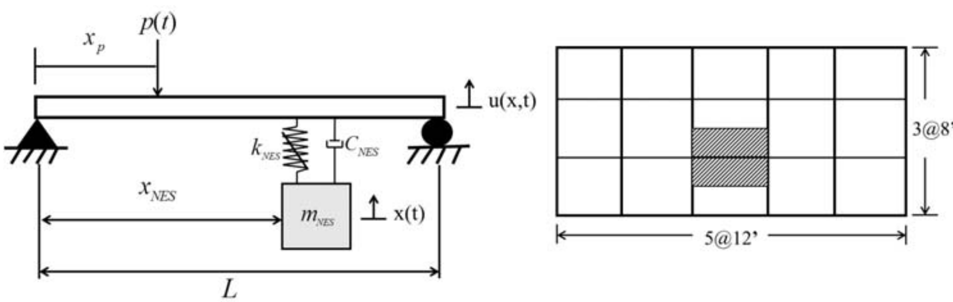

Figure 4 Floor model coupled with GCNES device

A combination of the assumed modes method and Lagrange's equation was used to derive the system's equation of motion (EOM). The assumed modes method is used to represent the response of the beam as a combination of shape functions. The two shape functions used to evaluate the motion of this system are shown in Eq. 10. With these shape functions, Lagrange's equation is used to derive the system's EOM. An eigenvalue analysis was carried out to determine the model's natural frequencies. The resulting natural frequencies of this system with no passive control device added are 5.23 Hz (32.9 rad/sec) and 20.91 Hz (131.4 rad/sec).

$$\begin{aligned}\Psi_1 &= \sin\left(\frac{\pi x}{L}\right)\\ \Psi_2 &= \sin\left(\frac{2\pi x}{L}\right)\end{aligned} \tag{10}$$

4. OPTIMIZATION

The system's dynamic analysis is carried out using MATLAB. The system's state space formulation is derived from the Lagrange EOM and modelled in Simulink. Optimization of both the TMD and NES cases are carried out using parameter sweeps. The loading considered in this optimization is a harmonic of amplitude 38.1 lb at 33 rad/sec (5.25 Hz). The objective function considered is the minimization of the RMS response, as measured from 25 sec to 30 sec of the response. This time frame was chosen in this evaluation in order to focus the optimization on the steady state response of the system. Contour plots from this optimization are shown in Fig. 5. The resulting optimized parameters are $k_{TMD} = 28650\,\text{lb/in}$ and $c_{TMD} = 0.1\,\text{lb-sec/in}$ for the TMD and $k_{NES} = 2.51\text{x}10^{10}\ \text{lb/in}^3$ and $c_{NES} = 7.5\,\text{lb-sec/in}$ for the GCNES.

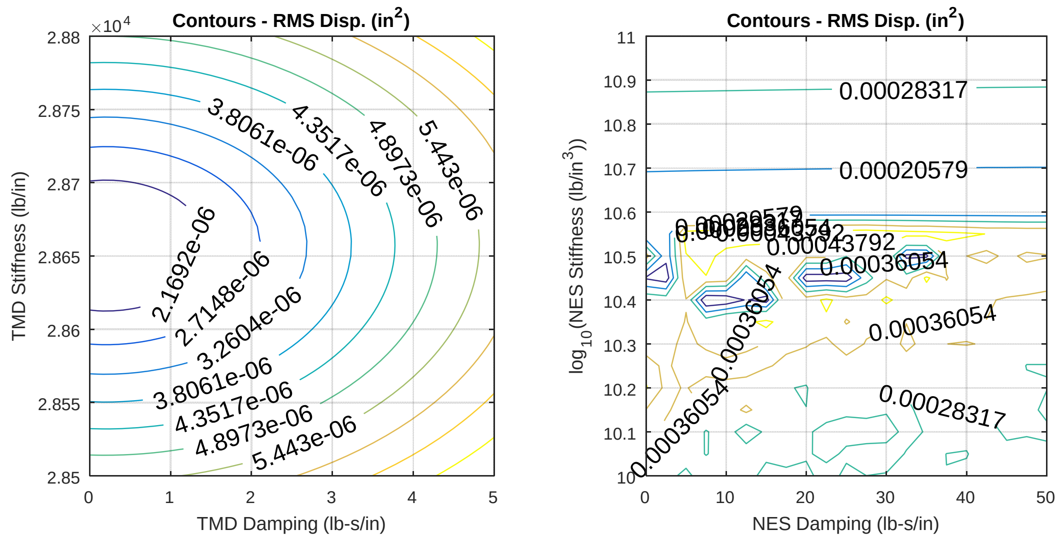

Figure 5 Contour plots of the TMD (Left) and NES (Right) used in the parameter sweep optimization.

5. REALIZATION OF OPTIMIZED GCNES

Using the previously obtained optimum parameters and an NES mass equal to five percent of the total floor mass, the physical properties of the GCNES device can be realized. Using Eq. 7, Eq. 8, Eq. 9, and setting a=0.0055 ft, the unknown geometric parameters L_0, h, k_1, and k_2 can be solved for simultaneously. These values are used to compare the cubic approximation restoring force (Eq. 6) with the complete restoring force function (Eq. 5). Fig. 6 shows a plot comparing the Taylor series approximate restoring force and the full restoring force with values obtained from solving the system of equations. One can see that the dynamically cubic approximation and the actual restoring force match closely over the displacement field.

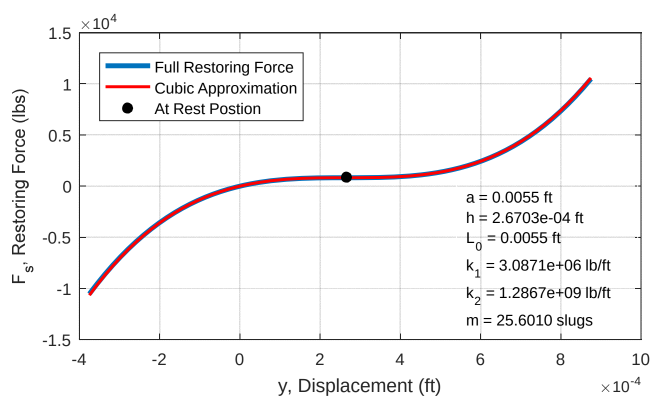

Figure 6 Restoring force comparison with realized physical parameters

6. RESULTS

The loading which was used in the optimization of the GCNES and TMD parameters (a 38.1 lb amplitude harmonic at 33 rad/sec) excites the first natural frequency of the floor model and results in a maximum steady state displacement of 0.003 in of the uncontrolled floor model at the measurement point considered. The steady state displacement of the floor with the TMD attached, GCNES attached, and with no passive control device is shown in Fig. 7. These results demonstrate that the optimized TMD and the GCNES are both capable of greatly reducing the steady state resonate response of the system; however, the TMD is able to control this response to a modestly smaller value.

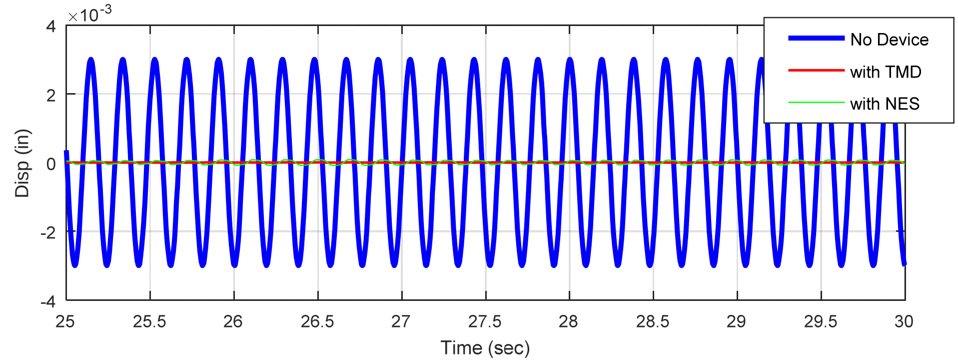

Figure 7 Steady state response to harmonic input at 33 rad/sec and scaled to result in 0.003 in maximum displacement

When the frequency of the loading is changed to 45 rad/sec, but the amplitude is kept at 38.1 lbs, the amplitude of the response decreases because the loading is no longer resonating with the first mode. The steady state response of the system with this loading on it is shown in Fig. 8. With this load, the TMD and the GCNES are both ineffective at controlling the vibration of the floor.

Fig. 9 shows the response of the system when the amplitude of the loading is increased such that the steady state maximum displacement of the floor is 0.003 in (the level used in the optimization), but the frequency of the loading remains at 45 rad/sec. As expected, since it is a linear device, the effectiveness of the TMD does not change when the amplitude of the loading is increased. However, this response does show that the GCNES is now moderately effective at reducing the steady state response to this loading.

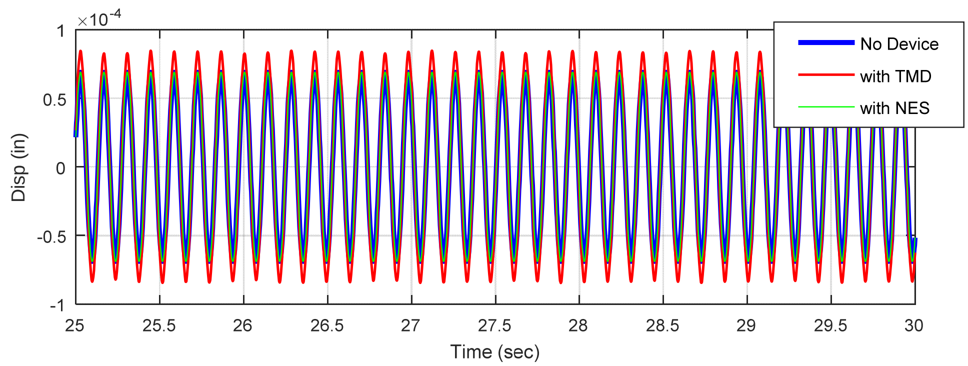

Figure 8 Steady state response to harmonic input of amplitude 38.1 lb at 45 rad/sec

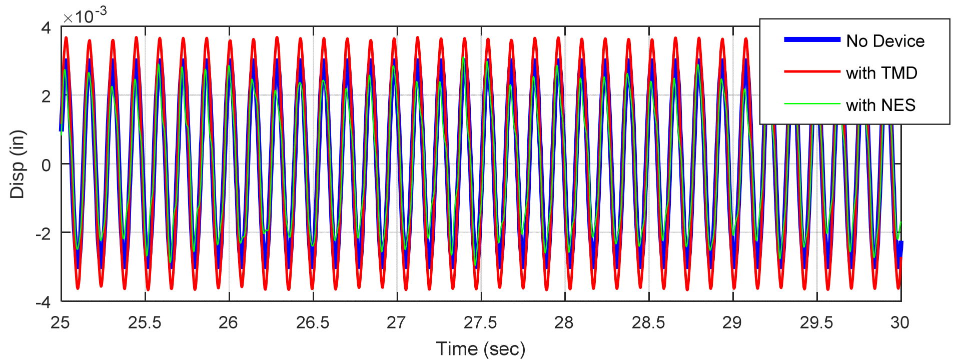

Figure 9 Steady state response to harmonic input at 45 rad/sec and scaled to result in 0.003 in maximum displacement

In a similar manner, Fig. 10 and Fig. 11 show the steady state response of the system to a load at 75 rad/sec with an amplitude of 38.1 lb and an amplitude increased to result in a maximum displacement of 0.003 rad/sec. These responses show that the TMD is ineffective at controlling the steady state response of the system at this input frequency; however, the GCNES can be effective at dramatically reducing the steady state response of the system when the response of the system is increased to the level the GCNES optimization occurred at.

The differences in the response of the system in Fig. 10 and Fig. 11, as well as Fig. 8 and Fig. 9, are due to the fact that the GCNES is not a linear device; thus, its performance is dependent on the amplitude of the loading and the response of the floor. Depending on the energy level of the response of the flooring system, the GCNES can resonate with any frequency the floor may respond at. The results presented here demonstrate that when the response of the system is small, the GCNES is likely to be ineffective; however, when the response of the system approaches the amplitude it was originally optimized at, the GCNES can be effective at mitigating the response of the system at resonant and non-resonant input frequencies. The complete range of frequency, energy, and amplitude levels the GCNES is effective over should be further studied.

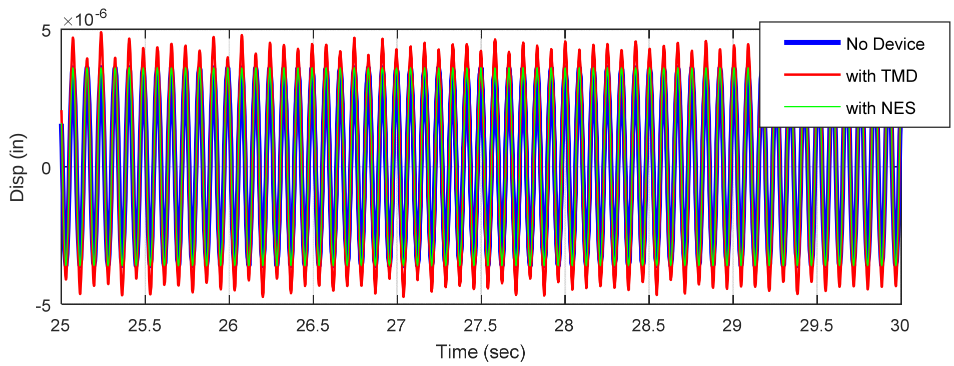

Figure 10 Steady state response to harmonic input of amplitude 38.1 lb at 75 rad/sec

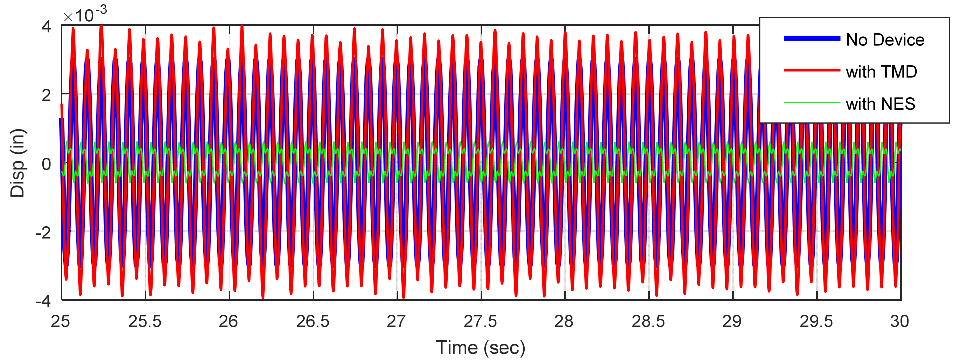

Figure 11 Steady state response to harmonic input at 75 rad/sec and scaled to result in 0.003 in maximum displacement

7. CONCLUSIONS

In this paper, a gravity compensated nonlinear energy sink (GCNES) designed to control vertical floor vibrations was proposed. The gravity compensation that is integral to this device is needed because the effect of gravity on the mass of a nonlinear energy sink (NES) would force it to vibrate about a displaced point with a non-zero tangential stiffness. With a non-zero tangential stiffness at its set point, the dynamics of the NES would fundamentally change. The GCNES's compensated nonlinear restoring force is achieved with a geometrically nonlinear combination of elastic oblique springs along with an inline spring and results in a device with a cubic dynamic restoring force. A simplified flooring system model was developed to examine the efficiency of the optimized GCNES in comparison with an optimized tuned mass damper (TMD). When excited at its resonance frequency, the TMD shows superior performance, but the GCNES is still capable of greatly reducing the response of the flooring system. When the frequency of the input was shifted away from the resonant frequency the devices were optimized, the TMD was shown to be ineffective at controlling the floor's vibration. In this situation, the performance of the GCNES was shown to be dependent on the amplitude of the response of the system and effective performance was demonstrated at some non-resonate frequencies when the amplitude of the response approaches what was considered in the GCNES's original optimization.

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